Constrained Optimization - Lagrange Multipliers Activity

Surface Activity: Let $z = f(x, y)$ represent the elevations in a mountainous area. Its contour plot is shown on page 2. Suppose you are taking a hike along a circular route. To visualize this, place the contour plot underneath your clear plastic surface so that the upper-left and lower-left corners of the contour plot match these edges of the top of the surface and the contour plot matches the surface above. Use the dryerase marker to trace the trail you would walk on this circular path along the mountain on the surface. (Helpful hint: It is easiest to stand above the surface, close one eye, and look straight down to draw the circular path).

Observe this path you drew through this mountainous terrain. On the surface, mark a large dot at all of the relative high points and low points you encountered along your hike. Then indicate with an "x" the absolute highest and lowest elevations that you encountered during your hike.

Let's observe a key relationship between the constraint curve and the contours. Calcplot3D Link (steps 1-3)

On the 2D contour plot, mark the points you found that were high points and low points on the surface (using the same symbols). Remember to stay on your circular hiking trail. (We will call these points **Lagrange points**: points on the constraint where a relative extremum may occur on the surface plot of *f*.)

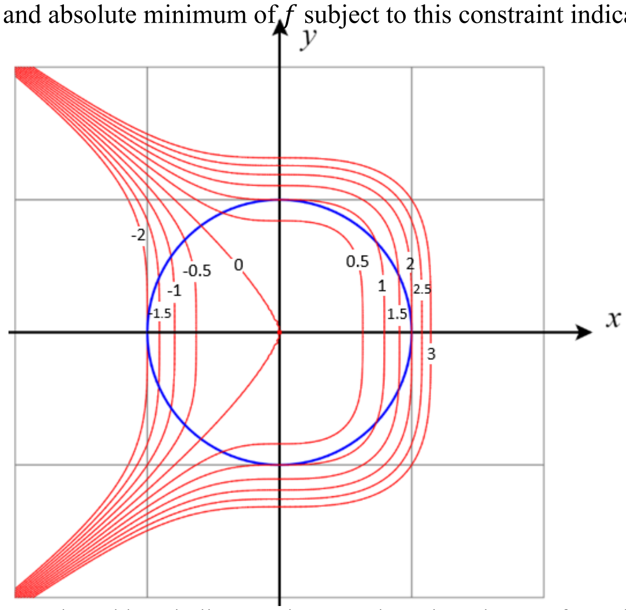
Discuss with your group and write down your conclusions:

1. What seems to be the relationship between the constraint curve and the contours of the function f at these low and high points? (It is clearer at two of the points than the others.) Why does this make sense? Explain.

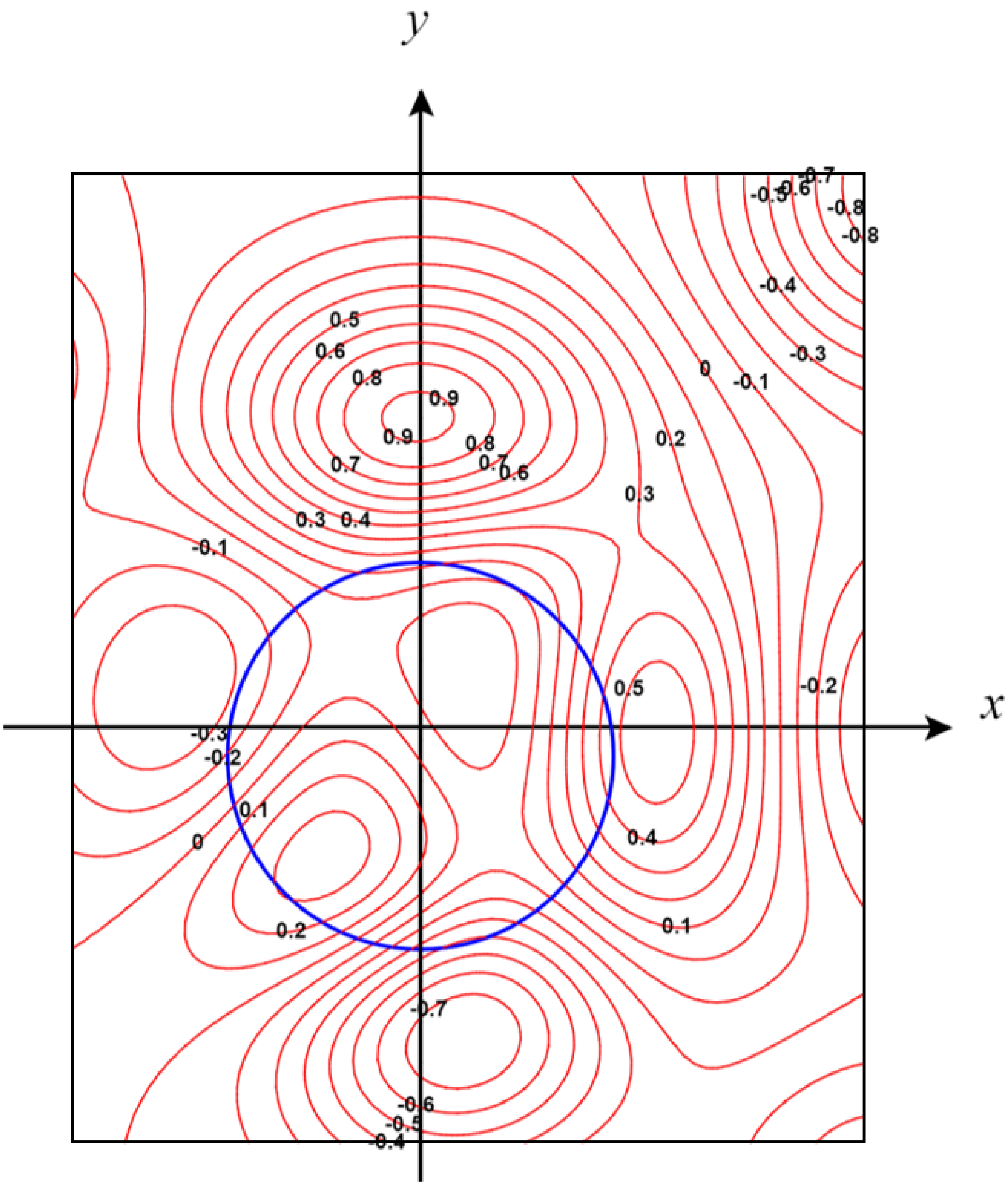
2. Could we identify all of these points just by looking at the constraint curve on the contour plot as if it were a trail map (without having seen the 3D surface)? If so, how? What would we be looking for?

3. Draw in (and label) a vector at one of these points to represent the direction of the gradient of *f* at the point. Let's consider the constraint curve to be a level curve of a second function of two variables. If we call this function *g*, draw in (and label) any vector(s) that represent(s) the direction of the gradient of *g* at this same point. What do you notice about the relationship between the gradient vectors of *f* and *g* here?

Practice Problem: Let's use what we just learned to determine the absolute maximum and minimum values of $f(x, y) = 2x^3 + y^4$, subject to the constraint of the unit circle. Start by drawing in all the "Lagrange points" on the contour plot of f below. Approximate the coordinates and function values at each of these points. Then state the absolute maximum and absolute minimum of f subject to this constraint indicating where each occurs.



CalcPlot3D Link (move the b slider)
CalcPlot3D Link



Developed by Shelby Stanhope and Paul Seeburger for Taking CalcPlot3D to the Next Dimension, NSF – IUSE #2121152
These materials can be accessed at: https://sites.monroecc.edu/multivariablecalculus/3d-learning-activities/