Integral Review Activity

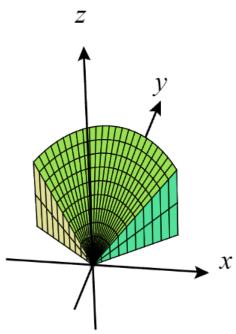
Part I: - Matching the 3D Solid to its Volume Integral

1. Match the 3D solid with the integral that computes the volume of that solid. Some of the shapes will match with more than one integral and some shapes will not be used.
2. Use the 3D table top axes to discuss the orientation of the object in 3-space during the matching process.
3. For the integrals in Cartesian/Rectangular and Cylindrical Coordinates, draw the “shadow region” on the axes provided in right-hand column of the table.
   ****Note: The radius values are NOT to scale … a larger object does not necessarily have a larger radius value.***

Color of the Solid	Integral	Shadow Region
	$\int_0^{2\pi}\int_3^5\int_{-\sqrt{25-r^2}}^{\sqrt{25-r^2}} r\,dzdrd\theta$	
	$\int_{-4}^{4}\int_0^{\sqrt{16-x^2}}\int_0^{y} 1\,dzdydx$	
	$\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_3^5 \rho^2\sin\phi\;d\rho d\theta d\phi$	
	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^5\int_{-5}^{-r} r\,dzdrd\theta$	
	$\int_0^4\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} y\,dxdy$	
	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^4 \rho^2\sin\phi\;d\rho d\theta d\phi$	
	$\int_0^{\frac{\pi}{4}}\int_{\pi}^{2\pi}\int_0^{\sqrt{2}} \rho^2\sin\phi\;d\rho d\theta d\phi$	
	$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^5 5\,r\,drd\theta - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^5 r^2\,drd\theta$	
	$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{0}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} 1\,dzdydx$	
	$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^{\pi}\int_3^5 \rho^2\sin\phi\;d\rho d\theta d\phi$	

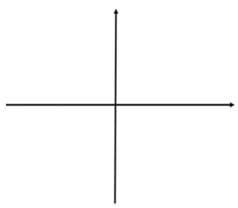
Part II: Integrals in various coordinate systems for a specific solid shape.

a) For the yellow solid, in the orientation pictured below, write the integral that finds the mass of the solid, given that the mass density function of the solid is $\rho(x, y) = x^2 + y^2$. Fully set up the integral both in rectangular coordinates and in cylindrical coordinates. Be sure to draw the shadow of the solid on the axes provided.
Note: The surfaces bounding the solid are $y = x,\ \ y = -x,\ \ z = y$ *and* $x^2 + y^2 = 9$.



Rectangular Coordinates:

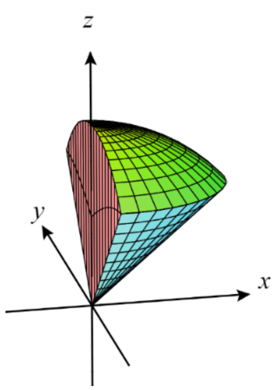
Shadow Region



Cylindrical Coordinates:

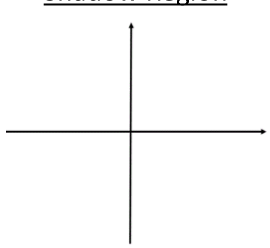
Which coordinate system is better for this problem? **Cartesian/Rectangular** or **Cylindrical** Why?

b) For the pink solid, in the orientation pictured below, fully set up the triple integral that finds the volume of the solid in cylindrical coordinates and in spherical coordinates. Be sure to draw the shadow of the solid on the axes provided. (Assume that the sphere surface has radius $\sqrt{2}$).



Cylindrical Coordinates:

Shadow Region



Spherical Coordinates:

Which coordinate system is better for this problem? **Cylindrical** or **Spherical** Why?