3D Surface Activity #1

Activity: Your group has a surface that is the graph of a function of two variables. The colored dot on the surface represents a point at which the elevation is zero. You also have a ruler and an erasable marker.

1. Using the ruler and marker, mark the location of a mathematical bug at some point on the surface with $z = 1$ cm. Now, trace a trail for the bug to follow so that the bug remains at the same elevation. If you find that your bug has fallen off the edge of the surface or is starting to loop back around and retrace its steps, is there another place on the surface with elevation $z = 1$, where you might start in order to walk a new path? Attempt to walk all possible paths with elevation $z = 1$.

2. Do the same for a mathematical bug at elevations $z = 2$ and $z = 0$.

3. Now do the same for two other mathematical bugs at elevations of your choice. At least one of the elevations should be negative.

4. Next, go to the board with your surface and draw these various *level curves* on a 2D graph. Be sure to label the *level curves* on your graph with the appropriate *z*-values. Check the work of another group.

Question: Have you seen a graph of lines/curves like this before? If so, where?

Concept Question: What do you think the term "level curve" means as it pertains to a function of two variables?

Instructor Demo: A 2D graph of the level curves of a surface is called a *contour plo*t. We can use CalcPlot3D to visualize contour plots.

5. Now use CalcPlot3D to check the accuracy of your *contour plot* (Note that level curves are also called the *contours* of a surface). Depending on whether your surface is "white, green, or yellow", the graph of your surface can be found in CalcPlot3D in the Menu →Examples → Function Surfaces→ Mountain White (Green, Yellow). Once you have pulled up the graph of your surface, generate a contour plot. Be sure to select thoughtful settings for your contours. This will depend on the actual *z*-values of the function, not the centimeter measurements we used above.

Question: Look at the steepest section on your surface. What corresponding feature(s) do you see on your contour plot? In general, how can we summarize how contour spacing relates to the slope of the surface?

Question: Pick a point on your $z = 0$ level curve and assume the mathematical bug is at that point on the surface. Draw a vector at the point on the contour map on the board (not on your surface) that indicates the direction the bug should step in order to go uphill most rapidly at that point. Repeat this process for at least two additional points on this contour. What is the relationship between the vectors you drew and the level curve $z = 0$?

Question: Suppose a company determines its cost for producing a particular good based on the expenses of labor and materials. Let the function $z = C(x, y)$ represent the cost of producing the good, where *x* is the expense due to labor and *y* is the expense due to materials. Describe what the level curves of this function represent.