May 2018 Puzzle

To the right are three identical dice. However, these are not standard dice as opposite faces do not total 7. Each of the numbers from 1 to 6 appears on exactly one face of each die and the touching faces of the dice have the same number. Indicate the arrangement of the die faces by labeling the faces in the "flattened view" shown below. To get things started, the face corresponding to 1 has already been labeled.

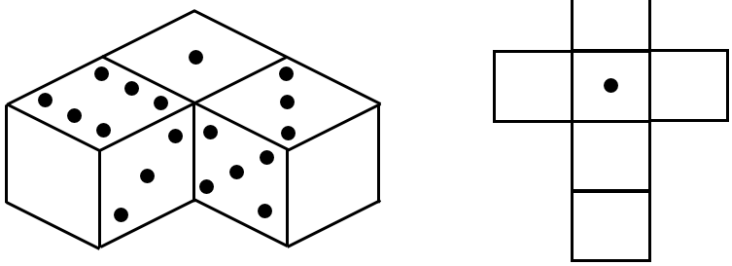
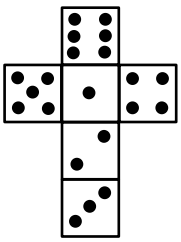


Figure 1

May 2018 Solution

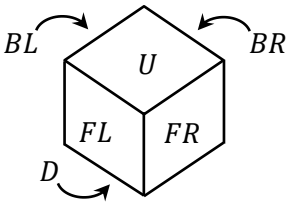
Answer: There are various ways to display the solution. The solution given to the right shows how the flattened view would look if the face containing 6 were placed above the face containing 1.



In order to explain how this solution can be obtained, we introduce the following naming convention to identify different faces of each of the three dice. First we will identify the leftmost die as X, the die in the middle as Y, and the rightmost die as Z. With the dice oriented as they are in the figure 1 above, we identify the faces as follows.

$U =$ top (up) face; $D =$ bottom (down) face; $FL =$ front-left face;

$FR =$ front-right face; $BL =$ back-left face; $BR =$ back-right face



So for example, in the figure 1 we have:

For X: $U = 6$ and $FR = 3$; For Y: $U = 1$; For Z: $U = 3$ and $FL = 5$

Looking at die X, we see that the face labeled 3 is adjacent to the face labeled 6 and the orientation of the dots are as shown in figure 2(a). Therefore, die Z must also have the face labeled 3 adjacent to the face labeled 6 with the dots in the same configuration. On die Z, this only allows for either $FR = 6$ or $BL = 6$. Otherwise, $BR = 6$ and the dots would be configured as shown in figure 2(b) or 2(c), which would make die Z different from die X.

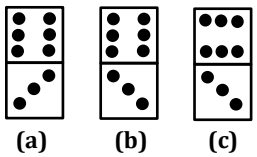
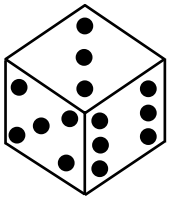


Figure 2

Suppose for the moment on die Z, that $FR = 6$ as shown to the right. This shows the faces labled 3, 5, and 6 all sharing a common corner. For die X have these same faces sharing a corner and with the dots in the same configuration, we must have $BR = 5$ on die X. Since face BR of die X is touching face FL of die Y, we must



have $FL = 5$ on die Y. This then implies that the face labeled 1 is adjacent to the face labeled 5. Therefore on die X, the only remaining locations for 1 to go would be on face D or BL. But opposite faces cannot total 7, and therefore on die X, $D \neq 1$ and we must have $BL = 1$. Continuing to look at die X, we only have two faces that haven't been determined yet—faces FL and D which must be labled with 2 and 4 in some order. We cannot have $FL = 2$, otherwise we would have 2 on the opposite face of 5 giving a total of 7 which is not allowed. We must therefore have $FL = 4$ and $D = 2$. This completely determines the following face labels for each die:

Die X: $U = 6$; $D = 2$; $FL = 4$; $FR = 3$; $BL = 1$; $BR = 5$

Die Y: $U = 1$; $D = 3$; $FL = 5$; $FR = 2$; $BL = 6$; $BR = 4$

Die Z: $U = 3$; $D = 1$; $FL = 5$; $FR = 6$; $BL = 2$; $BR = 4$

One can verify that this labeling satisfies all conditions required in the problem and it is this labeling that we provided in our answer to the puzzle.

We originally claimed on die Z, that either $FR = 6$ or $BL = 6$. The above labeling was based on the assumption that $FR = 6$. If we instead suppose that $BL = 6$ on die Z, we will run into a problem in constructing identical die that satisfy all conditions of the problem. We will explain how this problem arises without providing as much detail as was done for constructing the labeling given above.

If $BL = 6$ on die Z, the orientation of the dots composing 3, 5 and 6 on die Z would require $FL = 5$ on die X. Continuing to look at die X, we would have three remaining faces to be determined. The face to be labeled 1 cannot be opposite the face labeled 6 or those opposites faces would have a total of 7. Additionally, the face labeled 1 cannot be touching die Y since faces that touch must have the same number and we already know that face U of die Y is labeled with 1. Therefore, on die X, we must have $BL = 1$. With sides D and BR of X left to be labeled with 2 and 4, we would have to have $D = 2$ and $BR = 4$ to avoid opposite sides having a sum of 7. That would complete our labeling of die X as follows:

Die X: $U = 6$; $D = 2$; $FL = 5$; $FR = 3$; $BL = 1$; $BR = 4$

If we then construct two identical dice, and place die Y next to die X as shown in figure 1, we will then run into trouble when we try to place die Z next to die Y. On die Z we would have to have $U = 3$ and $FL = 5$. With the current construction, on die Z, we would then have $BL = 6$ and that must be the same as FR of die Y which it will be touching. However, one die Y, we would have $FR = 2$, which is different from 6.

We conclude that the only way to construct dice satisfying all conditions of the problem is as given in our answer.