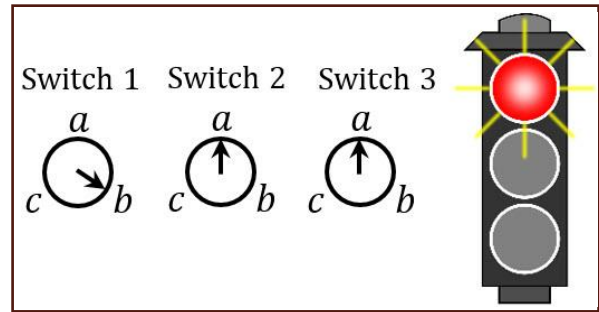


## December 2017 Puzzle

As shown in the figure to the right, there are three switches used to control a traffic light. Each switch has three possible settings of  $a$ ,  $b$ , or  $c$ , and the specific color (green, yellow, or red) of the traffic light is completely determined by the combined settings of the switches. Your job is to determine all of the settings of the switches that result in a green light based on the following information.



- Only one light is on for each of the settings.
- When all three switches are in the  $a$ -position (setting  $aaa$ ), the light is yellow.
- When switch 1 is in the  $b$ -position and switches 2 and 3 are in the  $a$ -position (setting  $baa$ ), the light is red.
- From any given setting, if you change the positions of all three switches, the light will change color.

## December 2017 Solution

The answer is:  $caa, cab, cac, cba, cbb, cbc, cca, ccb$ , and  $ccc$

To demonstrate this we use the table below. Each cell in the table represents a potential setting~color association. The number in each cell corresponds to the step in which that cell was determined to be a valid setting~color association or not.

Step 1: We start by placing a ① in the cell associated with  $aaa$ ~yellow to indicate that we have identified that the setting  $aaa$  results in a yellow light (this was a given). Since this implies that  $aaa$  does not result in green or red, we place a 1 in each of those corresponding cells to eliminate them from consideration.

Step 2: Since we know  $baa$  results in a red light (this was a given), we place a ② in the cell associated with  $baa$ ~red. This implies that  $baa$  does not result in green or yellow, and we place a 2 in each of those corresponding cells to eliminate them from consideration.

\*In the remaining steps we will repeatedly use the fact that changing the positions of all three switches will always change the light color.

Step 3: Since  $aaa$  results in yellow, the cells containing 3 can be eliminated.

Step 4: Since  $baa$  results in red, the cells containing 4 can be eliminated.

Step 5: At this point, we can conclude that settings *cbb*, *cbc*, *ccb*, and *ccc* all result in green. Therefore, we place a **5** in the corresponding cells.

Step 6: Since *cbb* results in green, the cells containing 6 can be eliminated.

Step 7: Since *cbc* results in green, the cells containing 7 can be eliminated.

Step 8: Since *ccb* results in green, the cells containing 8 can be eliminated.

Step 9: Since *ccc* results in green, the cells containing 9 can be eliminated.

Step 10: At this point we can conclude that the settings *abb*, *abc*, *acb*, and *acc* result in yellow and the settings *bbb*, *bbc*, *bcb*, and *bcc* result in red. Therefore, we place a **10** in the corresponding cells.

Step 11: Since *abb* results in yellow, the cells containing 11 can be eliminated.

Step 12: Since *abc* results in yellow, the cells containing 12 can be eliminated.

Step 13: Since *acb* results in yellow, the cells containing 13 can be eliminated.

Step 14: Since *bbb* results in red, the cells containing 14 can be eliminated.

Step 15: Since *bbc* results in red, the cells containing 15 can be eliminated.

Step 16: Since *bcb* results in red, the cells containing 16 can be eliminated.

Step 17: At this point only one possibility remains for each of the yet to be determined settings. We can now place a **17** in each of these cells to conclude our determination of the color of the light for each of the possible settings.

The completed table below shows that all settings beginning with *c* result in a green light.

Setting	G	Y	R	Setting	G	Y	R	Setting	G	Y	R
<i>aaa</i>	1	<b>1</b>	1	<i>baa</i>	2	2	<b>2</b>	<i>caa</i>	<b>17</b>	11	14
<i>aab</i>	7	<b>17</b>	15	<i>bab</i>	7	12	<b>17</b>	<i>cab</i>	<b>17</b>	12	15
<i>aac</i>	6	<b>17</b>	14	<i>bac</i>	6	11	<b>17</b>	<i>cac</i>	<b>17</b>	11	14
<i>aba</i>	8	<b>17</b>	16	<i>bba</i>	8	13	<b>17</b>	<i>cba</i>	<b>17</b>	13	16
<i>abb</i>	9	<b>10</b>	4	<i>bbb</i>	9	3	<b>10</b>	<i>cbb</i>	<b>5</b>	3	4
<i>abc</i>	8	<b>10</b>	4	<i>bbc</i>	8	3	<b>10</b>	<i>cbc</i>	<b>5</b>	3	4
<i>aca</i>	6	<b>17</b>	14	<i>bca</i>	6	11	<b>17</b>	<i>cca</i>	<b>17</b>	11	14
<i>acb</i>	7	<b>10</b>	4	<i>bcb</i>	7	3	<b>10</b>	<i>ccb</i>	<b>5</b>	3	4
<i>acc</i>	6	<b>10</b>	4	<i>bcc</i>	6	3	<b>10</b>	<i>ccc</i>	<b>5</b>	3	4