## September 2017 Puzzle

Place the letters $A, B$, and $C$ in the $4 x 4$ grid below so that each of the following "words" can be found in a row (read left to right) or column (read top to bottom).

ACAA BACA BACC BBAB BBAC CBBA CBCA CBBC


## September 2017 Solution

The two possible solutions to the puzzle are shown below.

| C | B | B | A |
| :---: | :---: | :---: | :---: |
| B | B | A | C |
| B | A | C | A |
| C | B | C | A |


| C | B | B | C |
| :---: | :---: | :---: | :---: |
| B | B | A | B |
| B | A | C | C |
| A | C | A | A |

As one can easily see, each of these solutions can be obtained from the other by simply interchanging each row with the corresponding column (change row 1 to column 1 , row 2 to column 2, etc...).

A strategic method for coming up with the solutions will now be provided.
We start by determining where the word $A C A A$ must go. Since we can easy convert a solution where $A C A A$ is in a row, to one where it is in a column (as described above), we will focus on a solution where $A C A A$ is in a column. This is the only word which has $A$ as its first letter and therefore, it cannot be placed in column 1 for otherwise, column 1 and row 1 would both have words starting with $A$. Similarly, $A C A A$ is the only word that has $C$ as its second letter, and therefore it cannot be placed in column 2. If $A C A A$ where in column 3, then there would be three rows containing words where $A$ was the third letter. However,
that isn't possible since there are only three words with $A$ as the third letter and $A C A A$ is one of them, which would have already been placed in column 3 . We conclude that $A C A A$ must be located in column 4.

Since no other words start with $A$, we see that no other $A$ 's can be placed in column 1 or row 1.

We now look to determine which word must be placed in column 1. The only word that starts with $B$ and ends with $A$, is $B A C A$ which can't be in row 1, since no other $A$ 's can be placed in row 1 . Therefore, row 1 cannot begin with a $B$. The only word starting with $C$ that doesn't contain the letter $A$ is $C B B C$, and thus $C B B C$ must be in the first column.

We immediately determine that BACA must be in row 3 as it is the only word starting with $B$ and ending with $A$.

Of the words remaining, only $B B A B$ and $B B A C$ have $A$ as the third letter and therefore these are the only two words to consider for column 2. But if we put $B B A C$ in column 2, then row 4 would start with $C C$ and there are no words that start $C C$. Thus, we must have $B B A B$ in column 2.

At this point, all that remains is to complete column 3. The only words remaining with $C$ as the third letter are $B A C C$ and $C B C A$. But if we put $C B C A$ in column 3, then $C B C A$ would also appear as row 1. Therefore, we conclude that column 3 must be BACC and the puzzle is complete.


| C |  |  | A |
| :---: | :---: | :---: | :---: |
| B |  |  | C |
| B | A | C | A |
| C |  |  | A |


| C | B |  | A |
| :---: | :---: | :---: | :---: |
| B | B |  | $C$ |
| B | A | C | A |
| C | B |  | $A$ |


| $C$ | $B$ | $B$ | $A$ |
| :---: | :---: | :---: | :---: |
| $B$ | $B$ | $A$ | $C$ |
| $B$ | $A$ | $C$ | $A$ |
| $C$ | $B$ | $C$ | $A$ |

