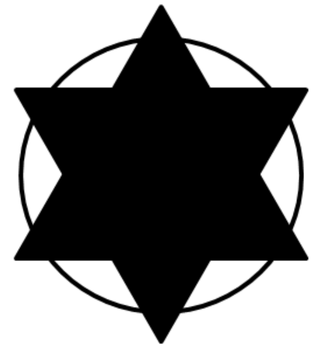
March 2016 Puzzle

The six-pointed star shown to the right has points formed from equilateral triangles with sides of length 1 inch. The star is placed on a circular region sharing the same center. Given that the area of the part of the star lying outside of the circle is the same as the area of the part of the circular region lying outside of the star, determine the radius of the circle.



Note: the circle shown in the figured does not necessarily satisfy the given conditions.

March 2016 Solution

This month we are allowing two solutions to the puzzle based on one's interpretation of the information provided.

If one interpreted the sides of the star as being 1 inch, then the radius of the circle is

$$\sqrt{\frac{\sqrt{3}}{\ }}$$

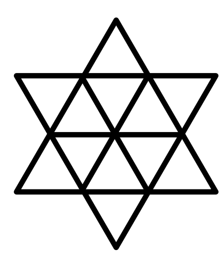
If one interpreted the sides of the star as being 1/3 inch (based on the star being formed by two larger equilateral triangles whose sides are 1 inch long), then the radius of the circle is

$$\sqrt{\frac{\ }{\sqrt{3}}}$$

We will provide details for the first interpretation; the result for the second interpretation can be obtained by dividing the result for the first interpretation by 3.

Because the area of the part of the star lying outside of the circle is the same as the area of the part of the circular region lying outside of the star, the areas of the circle and star must be equal to one another.

The figure to the right demonstrates that the area of the star is equal to 12 times the area of the equilateral triangles whose sides have length 1 inch. The area of an equilateral triangle with sides of length L is given by, $\frac{\sqrt{\ }}{\ }$. Therefore, the star and the circle have area, $\frac{\sqrt{\ }}{\ }(1)$ $\sqrt{\ }$ square inches. Since the area of a circle with radius r is given by , it follows that $\sqrt{\ }$. Solving for r we obtain, $\sqrt{\frac{\sqrt{3}}{\ }}$ as claimed.



rev. 10/2018